Section 5.3

The Indefinite Integral

(1) General Antiderivatives

(2) Antiderivative and the Indefinite Integral

(3) Calculating Indefinite Integrals

Antiderivatives

An **antiderivative** of a function $f(x)$ is a function $F(x)$ such that

$$F'(x) = f(x).$$

Calculating an antiderivative involves reversing the derivative process.

Theorem: If $g'(x) = f'(x)$, then $g(x) = f(x) + C$, where C is some constant.

This fact is a consequence of the Mean Value Theorem (see §4.3). In other words,

Theorem: If F and G are both antiderivatives of $f(x)$, then $G(x) = F(x) + C$, where C is some constant.

General Antiderivatives

Theorem: If F and G are both antiderivatives of $f(x)$, then $G(x) = F(x) + C$, where C is some constant.

In other words, a function $f(x)$ *has many different antiderivatives*, all of which differ by a constant.

$F(x) + C$ is called the **general antiderivative** of f.

For example, if an object's acceleration is $a(t)$, then

- the object's velocity is described by the general antiderivative $v(t) + C$,
- the object's displacement is described by $s(t) + Ct + D$, the general antiderivative of $v(t) + C$.

General Antiderivatives

Example 1: A guava moves along the x-axis with velocity $v(t) = 4t + 3$, starting at $t = 0$. What is its x-coordinate as a function of time?

Antiderivative Formulas (1)

Many formulas for antidifferentiation come from reversing the differentiation formulas that we we already know.

Function	x^n (where $n \neq -1$)	x^{-1}	e^x	a^x
Antiderivative	$\dfrac{x^{n+1}}{n+1} + C$	$\ln\lvert x\rvert + C$	$e^x + C$	$\dfrac{a^x}{\ln(a)} + C$

Antiderivative Formulas (2)

Function	$\cos(x)$	$\sin(x)$	$\sec^2(x)$	$\sec(x)\tan(x)$
Antiderivative	$\sin(x)+C$	$-\cos(x)+C$	$\tan(x)+C$	$\sec(x)+C$

Function	$\dfrac{1}{x^2+1}$	$\dfrac{1}{\sqrt{1-x^2}}$
Antiderivative	$\arctan(x)+C$	$\arcsin(x)+C$

However, this leaves many functions that we do not yet know how to antidifferentiate.

The Indefinite Integral

The notation

$$\int f(x)\,dx = F(x) + C$$

means that $F'(x) = f(x)$.

We say that $F(x) + C$ is the general antiderivative or the **indefinite integral** of $f(x)$.

Constant and Sum/Difference Rules

Let $F(x)$ and $G(x)$ be antiderivatives for $f(x)$ and $g(x)$.
Let c be a constant.

- $cF(x)$ is an antiderivative for $cf(x)$.
- $(F \pm G)(x)$ is an antiderivative for $(f \pm g)(x)$.

There do **NOT** exist similar rules for products, quotients, and compositions.

Calculating antiderivatives is significantly more complicated than calculating derivatives!

Though we don't easily obtain antiderivative rules for products and compositions, we will develop techniques in chapters 5 and 6 that will enable us to calculate antiderivatives of certain products and compositions.

General Antiderivatives: Examples

Example 2: Find the general antiderivative of each function.

(i) $f(x) = \frac{1}{2}x^2 - 2x + 6$

(ii) $g(x) = (x+5)(2x-6)$

General Antiderivatives: Examples

Example 2: Find the general antiderivative of each function.

(iii) $h(t) = \dfrac{3 + t + t^2}{\sqrt{t}}$

(iv) $p(x) = |x|$

Example 3: A particle is moving along the x-axis. In each case, find the position function $s(t)$ of the particle.

(a) The velocity function is $v(t) = 3^t$ ft/sec, and $s(2) = 9$.

(b) Acceleration is $a(t) = 2t + 5$, initial velocity is -5 ft/s, and $s(1) = 2$.

Example 4: A watermelon is dropped off a cliff and hits the ground with a speed of 112 ft/s. What is the height of the cliff? (Use 32 ft/s^2 for the acceleration due to gravity, and assume no wind resistance.)